

MATHEMATICS

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The translation of this course is based upon work supported by the grant no. CNFIS-FDI-2017-0056, “Internationalization of education and scientific research – a strategic tool for increasing visibility and improving quality at “Petru Maior” University of Tîrgu Mureş” – INTECS-IS.

Tîrgu Mureş
2017

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Chapter 1

Introduction to mathematical analysis. Single variable calculus

This Chapter presents single variable calculus and covers the properties of single variable functions and a brief presentation of series. Calculus is fundamental to almost all scientific fields including economics, management.

Single variable calculus, classical mathematical analysis, is the branch of mathematics dealing with limits and related theories. In the development of this fundamental branch of science a milestone was presented by the discovery of differentiation and integration calculus with more than 300 years ago. Between mathematicians, who have deep contribution in the development of this field, can be considered Newton, Leibniz, Euler, Cauchy, Riemann, Weierstrass, Cantor, Hilbert, Lebesgue, Banach.

In the last century were clarified notions such as: limit, continuity, integrability and was formulated the main objective of mathematical analysis as the study of the properties of the functions.

1.1 Introductory notes in Topology. Topologies on \mathbb{R}

In this chapter will be presented the single variable calculus, covering the differentiation and integration of functions of one variables and will conclude with a brief presentation of the infinite series. In this respect the basic concepts of the topology will be presented on real line and not will be used the concepts of general topology on abstract spaces. Calculus is fundamental in economics.

1.1.1 The neighbourhood of a point. The real line

Definition 1.1.1.

Let x_0 be a point on the line. We say *neighbourhood of x_0* a set V which contain an open interval (a, b) which contain the point x_0 . Thus, we can write: $x_0 \in (a, b) \subset V$.